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New Math Standards: What are the implications for instruction?

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FOCUS

COHERENCE

RIGOR

3600 seconds from now..

You will:

- Understand the background and overview of the Common Core State Standards for Mathematics
- Interpret the balance between math content and expectations for student behavior of mathematical practices
- Ponder your next steps supporting CCSSM

Understanding the background and overview of the Common Core State Standards for Mathematics

CCSS – A State-Led Initiative

- Beginning in the spring of 2009, Governors and state commissioners of education from 48 states, 2 territories and the District of Columbia committed to developing a common core of state K-12 English-language arts (ELA) and mathematics standards.

- The Common Core State Standards Initiative (CCSSI) was a state-led effort coordinated by the
  - **National Governors Association** (NGA) and
  - **Council of Chief State School Officers** (CCSSO).

- www.corestandards.org

The Challenge

"Every student must graduate ready for further education and the workforce. We must align our efforts so all our students are prepared to succeed in college or a career."

Tony Evers, State Superintendent
Wisconsin Department of Public Instruction

The CCSSM call for **significant** changes in mathematics teaching and learning to meet this challenge.

WISCONSIN DEPARTMENT OF PUBLIC INSTRUCTION

CCSSM: Getting it straight

- States led initiative – led by NGA & CCSSO
- Research-based
- Internationally benchmarked
- Widely reviewed
- Standards count on local implementation efforts
- More focused than previous standards
- Anchored in college and career readiness

Feedback and Review Teams

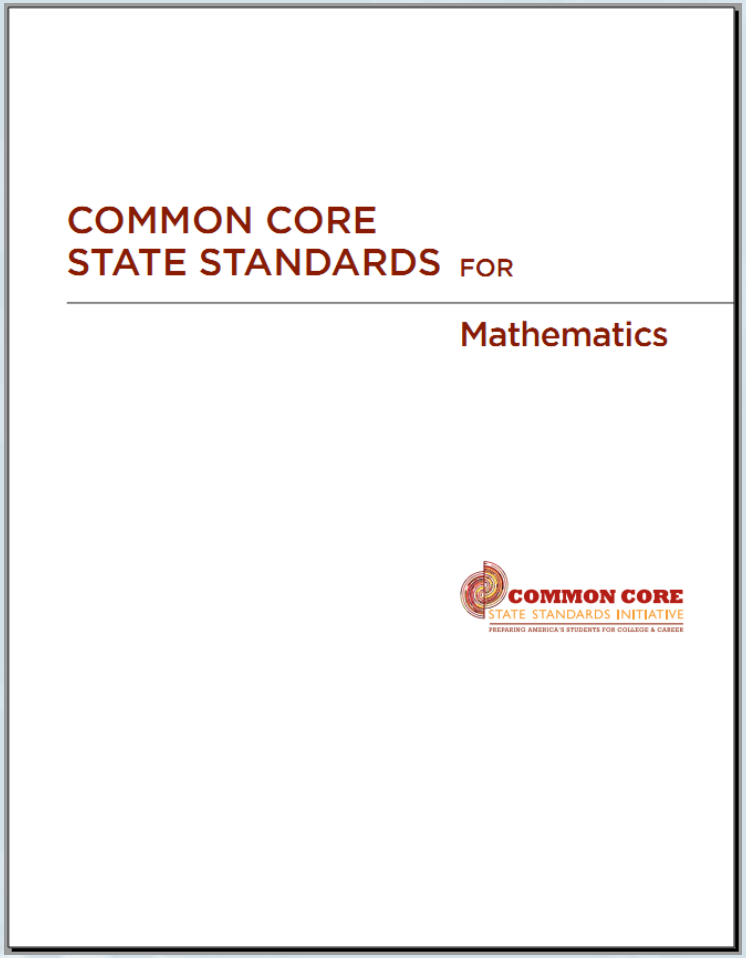
Mathematics feedback and review teams included:

K-12 teachers, postsecondary faculty, state curriculum and assessment experts, researchers, national organizations, including (but not limited to):

- National Council of Teachers of Mathematics (NCTM)
- National Education Association (NEA)
- American Federation of Teachers (AFT)
- Mathematical Association of America (MAA)
- American Council on Education (ACE)
- Association of State Supervisors of Mathematics (ASSM)
- Campaign for High School Equity (CHSE)
- Conference Board of the Mathematical Sciences (CBMS)

The Three Shifts in Mathematics

- **Focus** strongly where the standards focus
- **Coherence**: Think across grades and link to major topics within & across grades
- **Rigor**: Require intensity – conceptual understanding, procedural skills & fluency, and applications



Shift One:

Focus strongly where the Standards focus

- Significantly narrow the scope of content and deepen how time and energy is spent in the math classroom
- Focus deeply only on what is emphasized in the standards, so that students gain strong foundations

Critical CCSSM Structure

The CCSSM is constructed so that a standard is presented once.

After that, the standard is expected to be used, practiced & connected in building subsequent math & applications.

Grade Level Overview

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000

Format of K-8 Standards

Grade Level

Domain

Operations and Algebraic Thinking 1.OA

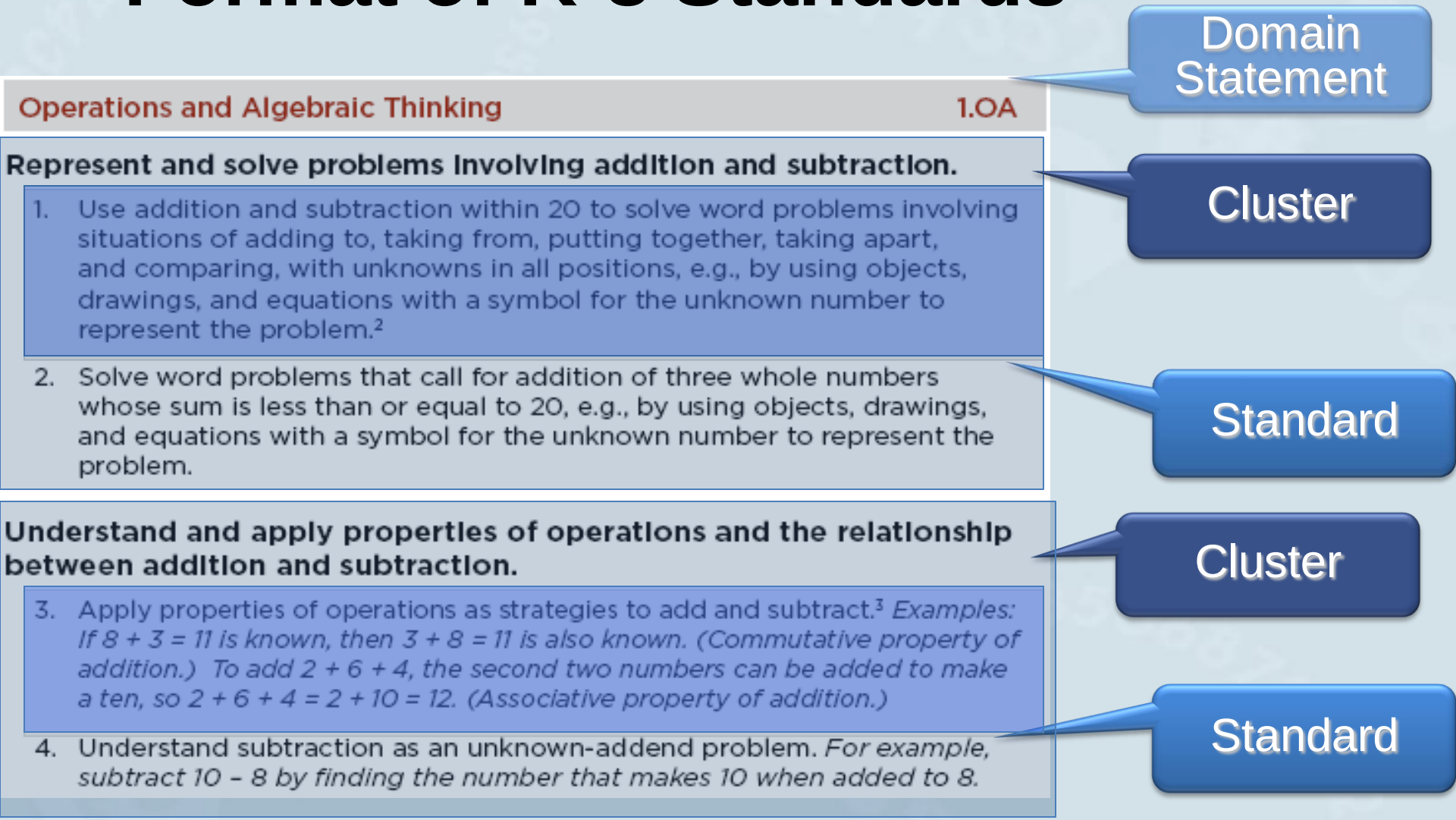
Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]
2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Format of K-8 Standards



Standards

Cluster

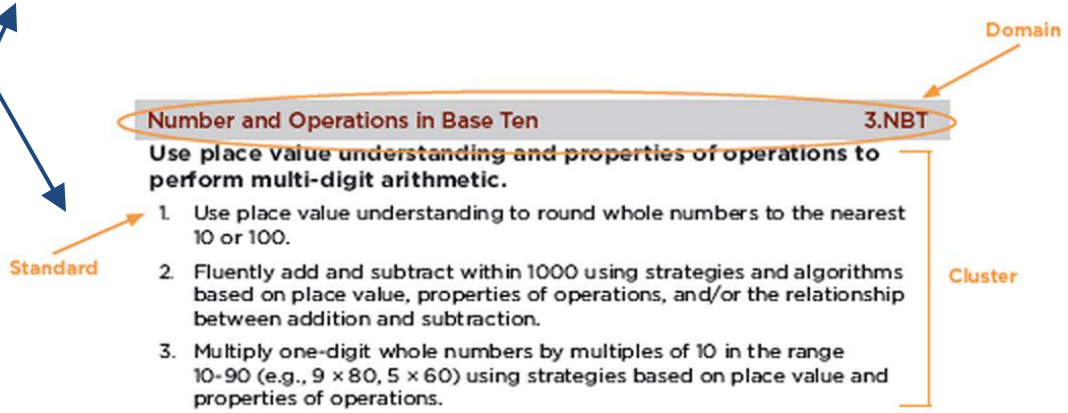
Domain

Mathematics » Introduction » How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters summarize groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.



These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know A should next come to learn B." But at present this approach is unrealistic—not least

Cluster Headings & Standards

- Cluster headings usually serve to **communicate the larger intent** of a group of standards.

- Individual standards in a cluster often **pinpoint some signs of success** in the endeavor, but the important endeavor itself is stated directly in the cluster heading.

- SBAC assessment design **focuses on cluster statements** – not individual standards

CCSS Assessments Will **Focus** Strongly on the Major Work of Each Grade

SMARTER
Balanced Assessment Consortium

prerequisite study for future grades' learning. A quality assessment should strive to reinforce focus and coherence at each grade level by testing for proficiency with central and pivotal mathematics rather than covering too many ideas superficially – a key point of the Common Core Standards.

PARCC Releases ITN To Develop Assessments

Design for focus and coherence. Consistent with the design of the CCSSM, the previous iteration of the assessment design adopted by the Leadership Team, and the extended discussion of emphases in the standards in the *PARCC Model Content Frameworks*, the Mathematics Assessment System as a whole and in each component will focus heavily on the major content[102] of each grade.

Interpret the balance between math content and expectations for student behavior of mathematical practices

CCSSM Implications for Teaching and Assessment

Teaching & Assessing through authentic connections of content and practices

"Designers of curricula, assessments, and professional development should all ***attend to the need to connect the mathematical practices to mathematical content in mathematics instruction****." (CCSSM, pg. 8)*

"The Common Core authors intended teachers and students to find 'points of intersection' between the mathematical content and practice standards..."

(Navigating the Mathematical Common Core Standards. p. 5)

"Students who do not have a balance of conceptual understanding and procedural understanding will not master the final expected outcomes."

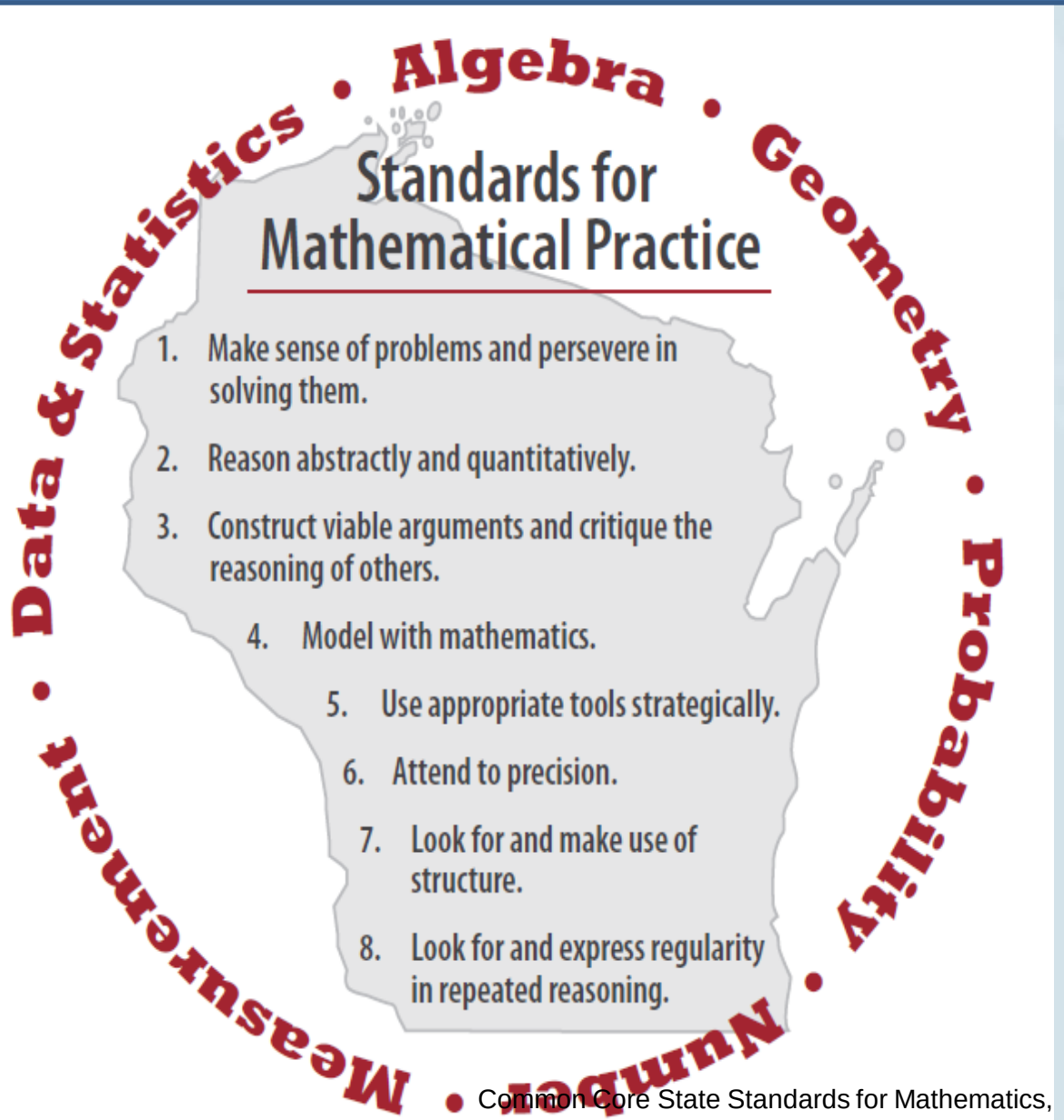
(Navigating the Mathematical Common Core Standards. p. 5)

Centering Mathematics in Wisconsin

for Mathematical Practice

describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

See Characteristics of Mathematically Proficient Students



Common Core State Standards for Mathematics, p. 6

1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematical Reasoning is a Refrain in the Content Standards

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Note generally such words as *justify* a conclusion, *prove* a statement, *explain* the mathematics; also *derive*, *assess*, *illustrate*, and *analyze.*

Standards for Mathematical Practice

- **Make sense of problems and persevere in solving them.** (MP 1)
- **Attend to precision.** (MP 6)

Reasoning and Explaining

- **Reason abstractly and quantitatively.** (MP 2)
- **Construct viable arguments and critique the reasoning of others.** (MP 3)

Modeling and Using Tools

- **Model with mathematics.** (MP 4)
- **Use appropriate tools strategically.** (MP 5)

Seeing Structure and Generalizing

- **Look for and make use of structure.** (MP 7)
- **Look for and express regularity in repeated reasoning.** (MP 8)

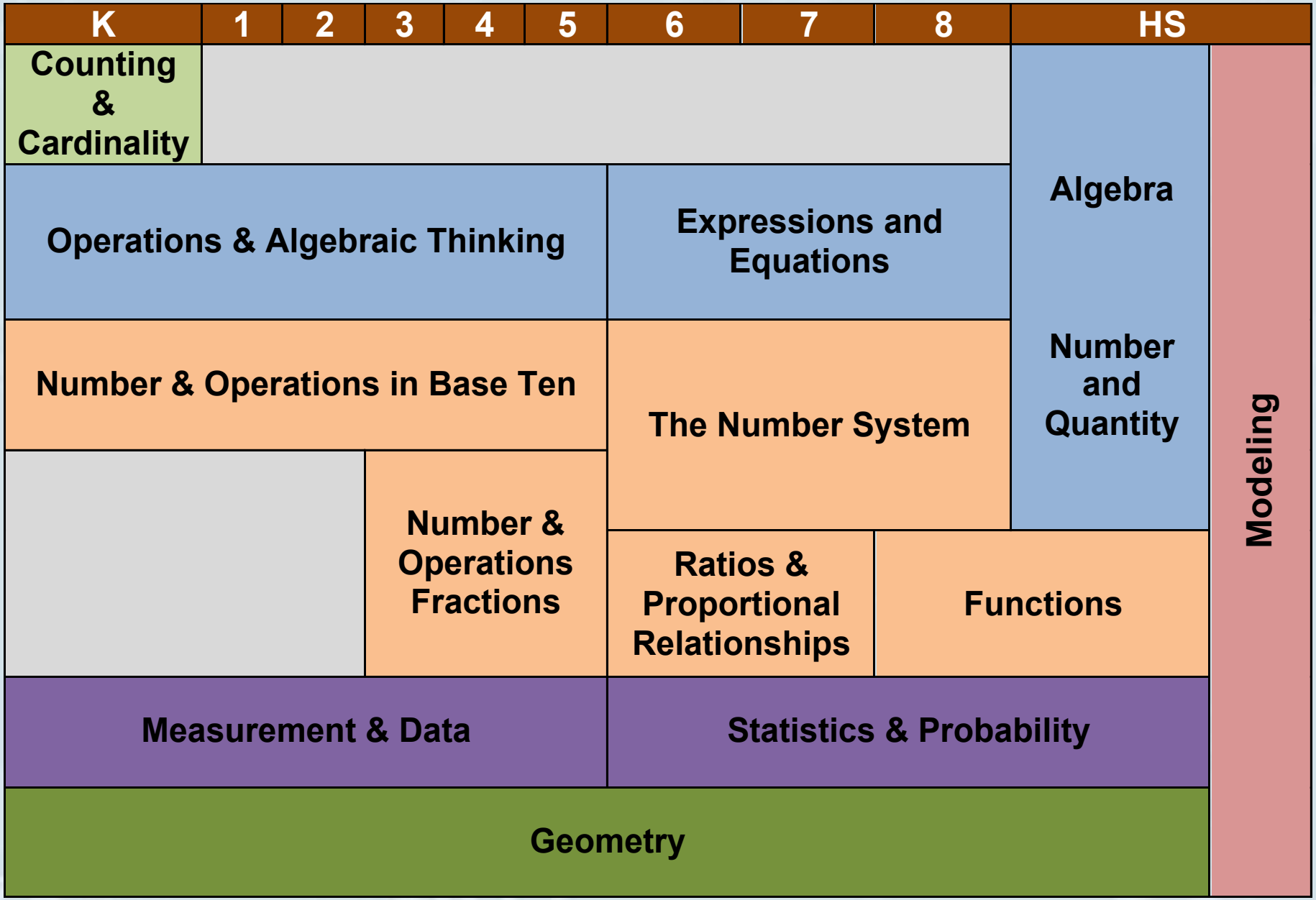
- **The *Standards of Mathematical Practice* are STANDARDS that the participating states have signed on to implement.**
  - CAUTION: Too many implementation and assessment projects are already starting to ignore or avoid the Standards for Mathematical Practice.
- **Placing attention and focus only on content standards is <u>insufficient</u>!**

Shift Two: **Coherence**

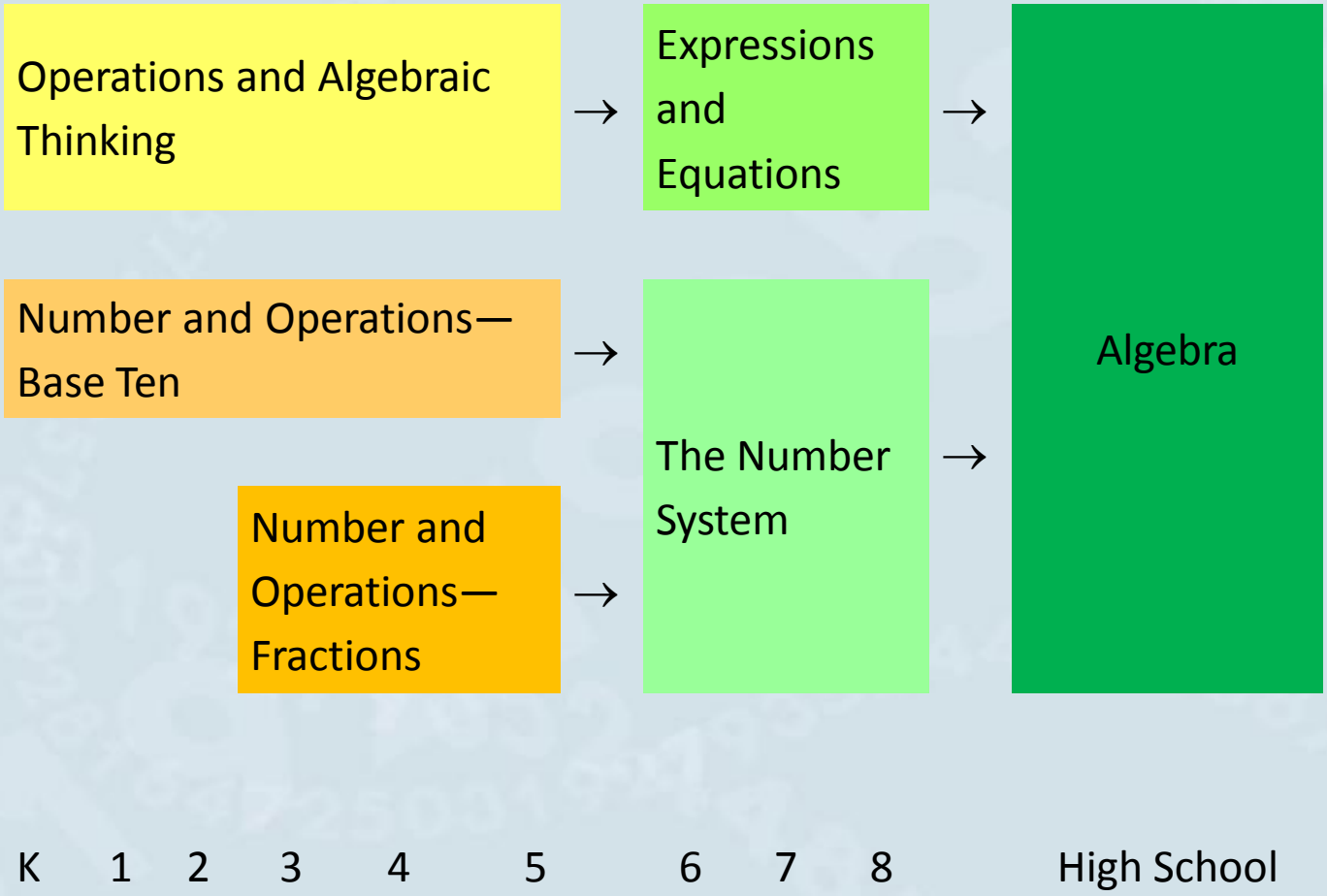
Think across grades, and link to major topics within grades

- Carefully connect the learning within and across grades so that students can build new understanding onto foundations built in previous years.
- Begin to count on solid conceptual understanding of core content and build on it. Each standard is not a new event, but an extension of previous learning.

K-8 Domains & HS Conceptual Categories



Focusing attention within Number and Operations



CCSSM and Implications for Teaching

Building understanding through context: Neighboring Grades and Progressions (from Zimba)

Certain cluster headings use language with a sense of motion from grade to grade. Some examples:

Grade 2 Work with equal groups of objects to **gain foundations for** multiplication.

Grade 4 **Generalize** place value understanding for multi-digit whole numbers.

Extend understanding of fraction equivalence & ordering.

Build fractions from unit fractions by **applying and extending previous understandings** of operations on whole numbers.

Grade 5 **Apply and extend previous understandings of** multiplication and division **to** multiply and divide fractions.

Coherence

Progressions Documents for CCSSM

http://commoncoretools.me/category/progressions/

- Draft K–6 Progression on Geometry
- Draft K–5 Progression on Measurement and Data (measurement part)
- Draft K–5 progression on Measurement and Data (data part)
- Draft K–5 Progression on Number and Operations in Base Ten
- Draft K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking
- Draft 3–5 progression on Number and Operations—Fractions
- Draft 6–8 Progression on Statistics and Probability
- Draft 6–8 Progression on Expressions and Equations
- Draft 6–7 Progression on Ratios and Proportional Relationships
- Draft High School Progression on Statistics and Probability
- Draft High School Progression on Algebra
- Draft High School Progression on Functions

Ponder your next steps supporting conversation, analysis, implementation, and reflection of CCSSM

CCSSM Getting Started with Fractions

Fractions in Grade 1 (Geometry Domain)

1.G.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. **Understand for these examples that decomposing into more equal shares creates smaller shares.**

Fractions in Grade 2 (Geometry Domain)

2.G.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. **Recognize that equal shares of identical wholes need not have the same shape.**

Fractions in Grade 3 (Geometry Domain)

3.G.2. Partition **shapes** into parts with **equal areas**. Express the **area of each part** as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and **describe the area of each part as 1/4 of the area of the shape.**

Building on Equal Shares

Four children are sharing 7 candy bars.
How much candy will each child get if they share the candy bars equally?

- Use paper strips to solve the problem.
- Use the language of 1.G.3, 2.G.3, and 3.G.2, to describe the shares.

Features of Models

	What is the whole?	How are equal parts defined?	What does the fraction indicate?
Area Model	The whole is determined by the area of a defined region	Equal area	The part covered of whole unit area
Set Model	The whole is determined by definition (of what is in the set)	Equal number of objects	The count of objects in the subset of the defined set of objects.
Number Line	Unit of distance or length (continuous)	Equal distance	The location of a point in relation to the distance from zero with regard to the defined unit.

Shifts . . . Teaching

Strategies and sense-making before algorithms
→ Strategies based on properties of the operations
→ Algorithms culminate years of prior work

Increased emphasis on specific models
→ Number line model
→ Area model

Discrete to continuous quantities

Using a "unit fraction" approach
→ Understand and use unit fraction reasoning and language and expect it of our students

Grade 5 SBAC Performance Task

Tito ate one-fourth of a cheese pizza, three-eighths of a pepperoni pizza, and one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza and the other half of the mushroom pizza. All the pizzas were the same size.

Tito says he ate more pizza than Luis because Luis did not eat any pepperoni. Luis says they ate the same amount.

Who is correct? Show your mathematical reasoning.

Smarter Balanced Assessment Claims for Mathematics

Concepts and Procedures	"Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency."
Problem Solving	"Students can frame and solve a range of complex problems in pure and applied mathematics."
Communicating Reasoning	"Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others."
Data Analysis and Modeling	"Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems."

Shift Three: **Rigor**

Equal intensity in conceptual understanding, procedural skill/fluency, and application

- The CCSSM require a balance of:
  - Solid conceptual understanding
  - Procedural skill and fluency
  - Application of skills in problem solving situations
- This requires equal intensity in time, activities, and resources in pursuit of all three

Cognitive Rigor Matrix

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom, 2001)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	- Recall, locate basic facts, definitions, details, events			
Understand	- Select appropriate words for use when intended meaning is clearly evident	- Specify, explain relationships - summarize – identify central ideas	- Explain, generalize, or connect ideas using supporting evidence (quote, text evidence, example…)	- Explain how concepts or ideas specifically relate to other content domains or concepts
Apply	- Use language structure (pre/suffix) or word relationships (synonym/antonym) to determine meaning	– Use context to identify word meanings - Obtain and interpret information using text features	- Use concepts to solve non-routine problems	- Devise an approach among many alternatives to research a novel problem
Analyze	- Identify the kind of information contained in a graphic, table, visual, etc.	– Compare literary elements, facts, terms, events – Analyze format, organization, & text structures	- Analyze or interpret author's craft (e.g., literary devices, viewpoint, or potential bias) to critique a text	– Analyze multiple sources or texts - Analyze complex/ abstract themes
Evaluate			– Cite evidence and develop a logical argument for conjectures based on one text or problem	- Evaluate relevancy, accuracy, & completeness of information across texts/ sources
Create	- Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept	-Generate conjectures or hypotheses based on observations or prior knowledge and experience	-Develop a complex model for a given situation -Develop an alternative solution	-Synthesize information across multiple sources or texts -Articulate a new voice, alternate theme, new knowledge or perspective

This matrix from the *Smarter Balanced Content Specifications for Mathematics* draws from both Bloom's (revised) *Taxonomy of Educational Objectives* and Webb's *Depth-of-Knowledge Levels* below.

http://www.smarterbalanced.org/

MP1: Make sense of problems and persevere in solving them

Students:

- Seek and communicate entry points or representations for the problem
- Communicate observed relationships and constraints
- Build a solution plan on observed relationships
- Monitor and evaluate their work and may report a change of strategy or perspective
- In examining a proposed solution, asks, does this make sense?

Teacher:

- Monitors students' thinking and processes to provide scaffolding for students' conjectures and plans.
- In summary presentations, requires student justifications and reasonableness, and seeks alternative solutions

Smarter Balanced Assessment Consortium

CLAIM 1

Concepts and Procedures:

Students can explain and apply mathematical concepts and

interpret and carry out mathematical procedures with precision and fluency.

MP2: Reason Abstractly and Quantitatively

Students:

- Create a coherent representation of a problem identifying relationships among quantities and their units
- Translate a problem situation from an Investigation or Example into a mathematical sentence (decontextualize)
- In generating equivalent symbolic statements, flexibly use properties of operations.

Teacher:

- Helps students make connections between problem setting and symbolic representations.
- Probes students' relational thinking, attending to quantities, their units, and the context.

SBAC CLAIM 2 Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

- primarily assess the student‘s ability to identify the problem and to arrive at an acceptable solution.

A Plate of Cookies

There were 28 cookies on a plate.

Five children each ate one cookie.

Two children each ate 3 cookies.

One child ate 5 cookies.

The rest of the children each ate two cookies.

Then the plate was empty.

How many children ate two cookies?

Show your work.

MP.3: Construct viable arguments & critique reasoning of others

- **Understand & use stated assumptions, definitions, and previously established results** in constructing arguments
- **Make conjectures and build a logical progression of statements** to **explore the truth of their conjectures**
- **Justify their conclusions; communicate them to others**
- **Listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments**.

MP3: Construct Viable Arguments & Critique the Reasoning of Others

Students:

- While completing an investigation, analyze situations and make or explore conjectures
- Build a logical progression of statements to justify a conjecture or present a counterexample
- Listen to or read the arguments of others and ask questions for clarification
- While completing an investigation, analyze each others' arguments.
- Reason inductively about data presented in context, making plausible arguments - including explaining any data that may be considered outliers or "one offs "

Teacher:

- Engages students in developing an exploration or viable argument which may include graphs, diagrams, constructions.
- Engages students in recalling knowledge, using prior results, assumptions, and definitions in building explanations or arguments .

SBAC CLAIM 3

Communicating & Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Claim 3 ***Sale prices***

Max bought 2 items in a sale.
One item was 10% off.
One item was 20% off.
Max says he saved 15% altogether.
Is he right? Explain.

Test propositions or conjectures with specific examples.

Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Use the technique of breaking an argument into cases.

Distinguish correct logic or reasoning from that which is flawed, and - if there is a flaw in the argument, explain what it is.

MP4: Model with Mathematics

Students:

•Apply prior contextual and mathematical knowledge to solve real world problems

•Display relationships among important quantities using tools such as diagrams, graphs, tables, formulas.

•Makes sense of Investigations by exploring a simpler real-life scenario by making assumptions and using approximations

•Make sense of an answer according to the context of the problem

Teacher:

•During real world Investigations, engage students in recognizing important quantities and exploring ways to represent mathematical relationships

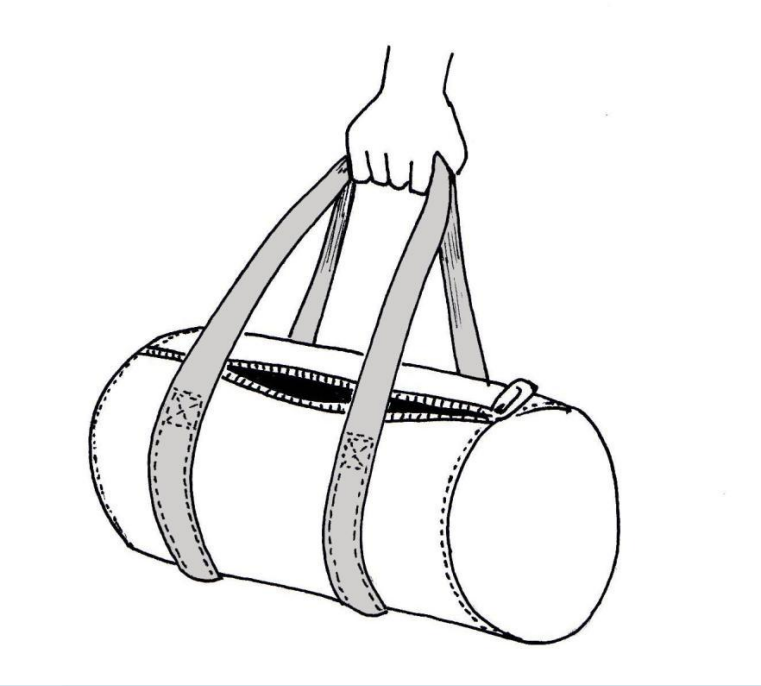
•Facilitates discourse around student conjectures about relations and arguments supporting varied modeling representations.

SBAC CLAIM 4

Modeling and Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

CR: Sports Bag



You have been asked to design
a sports bag.

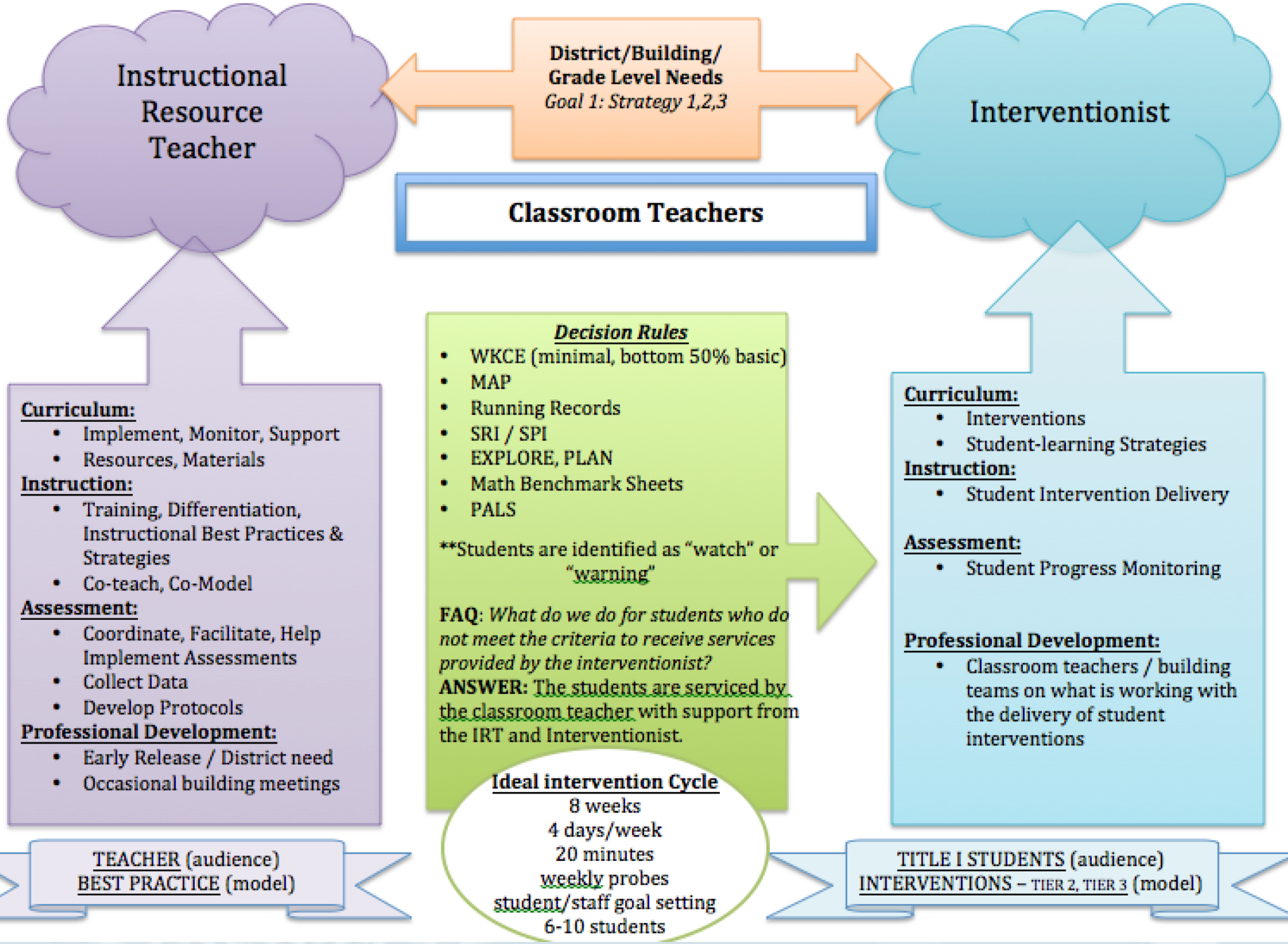
Requirements:
The length of the bag will be 60 cm.
The bag will have circular ends
of diameter 25 cm.
The main body of the bag will be made
from 3 pieces of material; a piece for the curved body, and the two circular end pieces.
Each piece will need to have an extra 2 cm all around it for a seam, so that the pieces may be stitched together.
.

1. **Make a sketch of the pieces you will need to cut out for the body of the bag. Your sketch does not have to be to scale. On your sketch, show all the measurements you will need.**

2. You are going to make one bag from a roll of cloth 1 meter wide. What is the shortest length that you need to cut from the roll for the bag? Describe, using words and sketches, how you arrive at your answer.

Content 7.G.4, 7.G.6; Practices P1, P4, P5, P6;
Claims 1, 2, 4.

School District of South Milwaukee IRT and Interventionist Model for Staff and Student Support



Resources/References

http://parcconline.org/parcc-content-frameworks

http://www.smarterbalanced.org/k-12-education/common-core-state-standards-tools-resources/

http://www.smarterbalanced.org/wordpress/wp-content/uploads/2011/12/MathContentSpecifications.pdf

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Inside Mathematics: www.insidemathematics.org

NCTM Illuminations: http://illuminations.nctm.org

http://commoncoretools.me

http://commoncoretools.me/category/progressions/

Math Assessment Project: http://map.mathshell.org/materials/index.php

www.corestandards.org

K-8 Publishers' Criteria for the Common Core State Standards for Mathematics.